Table 1 Values of exp(A/d)

	d		
A	2	4	6
0.50	1.284	1.133	1.087
0.4764	1.269	1.126	1.083
−0.282	0.868	0.932	0.954
−0.30	0.861	0.928	0.951
−0.50	0.779	0.882	0.920

It is clear that, because the index on the Reynolds number is small, this portion of the equation is not very sensitive either to the exact value of the index or the Reynolds number. Thus, in Eq. (1) for Re from 1 to 200, the value varies from 1 to 3.71; in Eq. (2), it varies from 1 to 2.33; and in Eq. (3), from 0.80 to 2.73. The exponential function is not particularly sensitive to the exact value of the constant given in Eqs. (1–3) for intersphere distances of 2–6, as shown in Table 1. Thus in Eq. (1) the value of the exponential factor is a little above unity, whereas in Eq. (3) it is a little below unity.

III. Proposed New Form of the Equations

By using a spreadsheet to calculate values of Eqs. (1–3) for $2 < Re < 200$ and $2 < d_{12}, d_{23} < 6$, I found that a close match could be provided by the following equations, which give a better sense of the relationships involved:

$$\alpha_1 = \frac{C_{D,1}}{C_{Ds}} = 1 - 0.1\frac{Re^{\frac{1}{4}}}{d_{12}} \exp\left(\frac{0.5}{d_{23}}\right) \tag{4}$$

$$\alpha_2 = \frac{C_{D,2}}{C_{Ds}} = 1 - 0.14Re^{\frac{1}{6}}\left(\frac{2}{\sqrt{d_{12}}} + \frac{1}{\sqrt{d_{23}}}\right) \tag{5}$$

$$\alpha_3 = \frac{C_{D,3}}{C_{Ds}} = 1 - 0.5\frac{Re^{\frac{1}{6}}}{\sqrt{d_{23}}} \exp\left(\frac{-0.5}{d_{12}}\right) \tag{6}$$

Values calculated from Eqs. (4) and (5) differed from those calculated from Eqs. (1) and (2) by, at most, 0.3 and 3%, respectively. Equation (6) gave results within 10% of Eq. (3) up to $Re = 100$, rather more for extreme combinations of d_{12} and d_{23} at $Re = 200$. For example, at $Re = 50$, $d_{12} = d_{23} = 4$; then, values from Eqs. (1), (2), and (3) are 0.925, 0.586, and 0.536, respectively; from Eqs. (4), (5), and (6), values are 0.925, 0.597, and 0.577, respectively. The above have been compared only with the curve-fitting equations of Ramachandran et al., not with their data. The constants proposed in this communication are to one or two significant figures. A better fit might be obtained by comparison with the original data, and a precision of two significant figures thereby justified.

IV. Interpretation of New Equations

The form of the equations may be related to the theory of wakes produced by Moore[2] for bubbles (although the slip conditions are different from rigid spheres). According to Moore, there is a region to the rear of a bubble in which the vorticity from the boundary layer is transferred to the wake. This has a linear size dependent upon $Re^{-1/6}$ and velocity difference from the potential flow dependent upon $Re^{-1/3}$. Moore comments that these powers may seem surprising, but seem to be attributable to the three-dimensional nature of the flow. No such powers would arise for flow past a two-dimensional bubble. A term $f(Re^{1/4}/d)$ is derived by Harper[3] from Moore's theory for drag resulting from the wake between two or more bubbles.

Equations (4) and (6) both show the end sphere experiencing drag reduction (Re and d factor) attributable to the adjacent sphere, which is modified by the presence of a third sphere (exponential factor varies around unity). Equation (5) shows that the center sphere is affected by both of the others, the effect of the leading sphere being twice that of the trailing sphere for the same intersphere distances.

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Reply to M. J. Pitt

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THE simplified drag equations for laminar flow past three closely spaced monodisperse spheres proposed by Pitt are appreciated because of their ease of use and formal interpretation for bubbly flows. Perhaps a limit of $Re \leq 100$ should be suggested because of possible nonlinear error accumulation at higher Reynolds numbers and large particle spacings.

Drag and Nusselt number correlations extended to spherical and nonspherical droplets with mass transfer (i.e., blowing) may be found in Refs. 1–4.

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